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MULTILEVEL METHODS FOR THE APPROXIMATION OF SINGULAR SOLUTIONS OF COMPLETELY CONTINUOUS OPERATOR EQUATIONS

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ABSTRACT. The direct techniques for approximating singular solutions of nonlinear equations depending on parameters transform the original problem into that of solving a suitable augmented system. Dealing with equations involving completely continuous operators, a multilevel approach to the solution of these larger systems is presented.

1. Introduction. We are concerned with the approximation of singular points in branches of solutions of nonlinear parameter-dependent equations having the form

$$F(u,\beta_1,\dots,\beta_{p-1},\gamma) = u - K(u,\beta_1,\dots,\beta_{p-1},\gamma) = 0, \tag{1.1}$$

where $u \in U$, with U a real Banach space, $\gamma \in \mathbf{R}$ and the β_i's are $(p-1)$-additional real parameters, for some $p \geq 1$. From now on, we assume that F is a C^ν-mapping, with $\nu \geq 3$, from an open set $D \subset U \times \mathbf{R}^p$ into U and that the operator K is completely continuous.

As is well known, the direct techniques for the approximation of a singular solution are based on the construction of an augmented system having it as a regular solution (cf. [**9**] for a general discussion). Then, moving from a suitable starting point, obtained for instance by a continuation procedure, the arising system is solved by an iterative method. In several practical cases, this procedure is carried out using operator approximations, on which the accuracy of the computed solution depends (cf. [**11, 20**]).

In this paper we deal with singular solutions of (1.1), such as turning points and, when unfolded, bifurcation points, which fall within the unifying theory recently developed by Griewank and Reddien in [**10, 11**], where the corresponding augmented system is built up through the minimum number of additional scalar equations characterizing the singularity. For solving this larger system, we present an approach based

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