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A FULLY-DISCRETE TRIGONOMETRIC COLLOCATION METHOD

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This work is dedicated to Prof. Dr. Dr. h.c. mult. W. Haack
on the occasion of his 90th birthday

ABSTRACT. An error analysis is given for a fully-discrete collocation method applied to periodic, elliptic pseudodifferential equations. The trial space consists of the trigonometric polynomials of degree n, and the method can be implemented efficiently using fast Fourier transform and multigrid techniques. If the order of the pseudodifferential operator is an integer, and if the exact solution is r times continuously differentiable, then the error in the maximum norm is $n^{-r} \log n$. This estimate is sharp, since it is of the same order as for the trigonometric interpolant. As applications, we consider Symm's integral equation on closed curves and open arcs.

1. Introduction. We investigate a fully-discrete collocation method for periodic, elliptic pseudodifferential equations, such as those arising from boundary integral equations on closed curves. The method uses equally-spaced collocation points, and a trial space consisting of trigonometric polynomials, just as in our earlier paper [**12**]. This approach leads to a very simple treatment of the principal part of the operator, leaving only a smoothing operator to be handled by a Nyström-like quadrature. Our aim here is to extend the analysis in [**12**] by taking account of the quadrature errors, and also to eliminate one factor of $\log n$ from the pointwise error estimates in the case when the order of the pseudodifferential operator is an integer.

We shall not discuss in detail the practical implementation of the method. The quadratures can be evaluated efficiently using the fast

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