JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 5, Number 1, Winter 1993

A GENERAL METHOD FOR SOLVING PLANE CRACK PROBLEMS

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0. Introduction. There have been many works on plane crack problems. Each of them has solved a particular problem for the case, either special in the location of the cracks and the interfaces or in the boundary conditions, for example, [**1, 2**]. In this paper a unified method of solution for such problems is proposed, which is effective in the following more general case. Assume there are a set of arc-wise smooth nonintersecting cracks in the composite media with certain interface. The cracks may touch or pass through the interface, or even lie on the interface. It reduces to a singular integral equation which is uniquely solvable under certain natural additional requirements for its solution. A new idea for determining the order of singularity of the solution at any node of the problem is suggested. Here, by a node of the problem, we mean either any tip or corner point of the cracks, any corner point of the interface, or any point of intersection of a crack and the interface.

For definiteness, we consider the first fundamental problems only (Muskhelishvili [**6**]) although our method is also effective for the second fundamental problems or mixed boundary problems. For simplicity, we assume the interface is a straight line. We shall illustrate our method for two somewhat special cases which often occur in practice, but the method is universally in effect for the general case.

1. Bonded half-planes with cracks. Assume an elastic infinite plane consists of two bonded half-planes, the upper half-plane Z^+ and the lower half-plane Z^-, and there are p cracks $\gamma_1, \dots, \gamma_p$ in the plane, some of which lie in Z^+ or in Z^- (maybe touch the x-axis) and the others locate on or pass through the x-axis. Assume each crack $\gamma_j = a_j b_j$ is an arc-wise Lyapunov arc: the angle of inclination $\theta(t)$ of the tangent at t on each of its smooth subarcs is Hölder continuous. Denote $\gamma = \sum_{j=0}^{p} \gamma_j$, $X = \{\text{the } x\text{-axis}\} \backslash \gamma$ (which is the interface) and

Received by the editors on July 21, 1992.