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THE h-p-VERSION OF SPLINE APPROXIMATION METHODS FOR MELLIN CONVOLUTION EQUATIONS

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ABSTRACT. We consider the numerical solution of Mellin convolution equations on an interval by the h-p-version of spline approximation methods. Using a geometric mesh refinement towards the singularity of the integral equation, we prove stability and exponential convergence in the L_q norm, $1 \leq q \leq \infty$, for Galerkin, collocation and Nyström methods based on piecewise polynomials.

1. Introduction. We consider the approximate solution of the one-dimensional Mellin convolution equation

$$u(x) - \int_0^1 \kappa(x/y)u(y)y^{-1}\,dy = f(x), \quad x \in I := (0,1), \tag{1.1}$$

where f and κ are given functions and u is the unknown function. Such integral equations having a fixed singularity at the point $x = 0$ arise in a variety of applications; for example, they occur when boundary integral methods are applied to potential problems in plane regions with corners or to crack problems in linear elasticity (see [**4, 13**] and the references therein). Note that the integral operator in (1.1) is not compact so that standard theories for the numerical analysis of second kind Fredholm integral equations cannot be applied. Nevertheless, using graded meshes and modified spline spaces, results on stability and optimal convergence orders of Galerkin, collocation and quadrature methods for Equation (1.1) which are based on piecewise polynomial basis functions have been obtained in [**4, 6, 7, 9**]. These papers apply the technique of the traditional h-version of spline approximation methods where accuracy is achieved by decreasing the mesh size h, while keeping the degree p of piecewise polynomials fixed.

In the present paper we study the h-p-version of those approximation methods which is obtained if one simultaneously refines the mesh and

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