# OPEN PROBLEMS
# IN SINGULAR INTEGRAL THEORY

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ABSTRACT. The following list of 19 problems were proposed by participants of the singular integral operators NSF-CBMS regional research conference, held at the University of Montana from August 26–September 1, 1989. The contributing participant is noted in parentheses at the beginning of each problem. A number of other open problems appear in the CBMS monograph *Singular Integral Operators* (to appear) detailing the contents of the lectures presented by Professor F. Michael Christ during this conference. It is hoped that the monograph and these open problems will serve to inspire continued work in this fascinating and useful area of mathematics. Symbols and concepts not defined in these problems are explained in the monograph or in the two books listed in the references.

**Open problems.**

1. (J. Alvarez). Consider the class $\mathcal{M}$ of operators $K$ acting on Schwartz functions:

$$(1) \qquad Kf = \int e^{-2\pi i x \circ \xi} \rho(x,\xi) \hat{f}(\xi) \, d\xi + Rf$$

where

$$(a) \qquad |D^\alpha x D_\xi^\beta \rho(x,\xi)| \le C_{\alpha,\beta}(1+|\xi|)^{-|\beta|+\delta|\alpha|},$$

$0 < \delta < 1$, $|\alpha| \le M$, $|\beta| \le N$, for some $M, N$, and

$$(b) \qquad R, R\frac{\partial}{\partial x_j}, \frac{\partial}{\partial x_j}R, \quad 1 \le j \le n, \quad \text{map } L^p \text{ into itself,}$$

for $p_0 \le p \le p_0'$, for some $1 < p_0 \le 2$. Then $\mathcal{M}$ is a self-adjoint Banach algebra with the norm

$$||K||_\mathcal{M} = \inf \left\{ \inf_{|\alpha| \le M, |\beta| \le N} C_{\alpha,\beta} + ||R||_{L^{p_0}} + ||R||_{L^{p_0'}} \right.$$

$$\left. + \sum_{j=1}^n \left( \left\| \frac{\partial}{\partial x_j}R \right\|_{L^{p_0}} + \left\| R\frac{\partial}{\partial x_j} \right\|_{L^{p_0}} + \left\| \frac{\partial}{\partial x_j}R \right\|_{L^{p_0'}} + \left\| R\frac{\partial}{\partial x_j} \right\|_{L^{p_0'}} \right) \right\},$$