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PHASE TRANSITION PROBLEMS IN MATERIALS WITH MEMORY

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1. Introduction. In this paper phase transition problems in materials with memory are formulated and studied. As usual for this kind of material, the classical Fourier conduction law is modified by adding a memory term to the heat flux. Also, since different phases are involved, the internal energy e is allowed to depend on the phase variable χ. By considering the standard equilibrium condition at the interface between two phases, we deal with the Stefan problem accounting for memory effects. Next, replacing this equilibrium condition by a relaxation dynamics, we represent superheating and supercooling phenomena. The application of a fixed point argument helps us to show the existence and uniqueness of the solution to the latter relaxed problem. Hence, taking the limit as a kinetic parameter goes to 0, we prove an existence result for the former Stefan problem. In this case the uniqueness is deduced by contradiction.

Let us now introduce and briefly discuss the models. In order to account for memory effects in heat conduction phenomena, some modifications of the classical Fourier law have been proposed along with different constitutive assumptions on the internal energy. Here we follow a well known and widely investigated theory (see, e.g., [**8**] and its references) to approach materials having a memory of the past histories. Let us consider a sample of such a material (supposed to be homogeneous and isotropic) located in a bounded domain $\Omega \subset \mathbf{R}^3$ at each point $x \in \Omega$ for each time $t \in \mathbf{R}$. According to Coleman and Gurtin [**4**] (see also [**2, 9**]) we assume that the following *linear non Fourier* law holds:

$$(1.1)\quad \mathbf{q}(x,t) = -k_o \nabla\vartheta(x,t) - \int_{-\infty}^{t} k(t-s)\nabla\vartheta(x,s)\,ds, \quad (x,t) \in \Omega\times\mathbf{R},$$

where $\mathbf{q} : \Omega \times \mathbf{R} \to \mathbf{R}^3$ represents the *heat flux*, $\vartheta : \Omega \times \mathbf{R} \to \mathbf{R}$ is the *absolute temperature*, and, as usual, ∇ denotes the gradient

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