JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 4, Number 4, Fall 1992

ON THE PIECEWISE CONSTANT COLLOCATION METHOD FOR MULTIDIMENSIONAL WEAKLY SINGULAR INTEGRAL EQUATIONS

G. VAINIKKO

ABSTRACT. Convergence rates of the piecewise constant collocation method (PCCM) and related methods for weakly singular integral equations on an open bounded set $G \subset \mathbf{R}^n$ are investigated in [**3, 7–10**]. The main purpose of this paper is to show how the l_h^2 elements of the system of PCCM can be evaluated in $\mathcal{O}(l_h^2)$ arithmetical operations with an accuracy preserving the convergence rate of the basic PCCM.

1. Integral equation. In this paper, we shall deal with an integral equation

$$u(x) = \int_G K(x,y)u(y)\,dy + f(x), \quad x \in G, \tag{1.1}$$

where $G \subset \mathbf{R}^n$ is an open bounded set with a piecewise smooth boundary ∂G. The following assumptions (A1)–(A4) are made.

(A1) The kernel $K(x,y)$ is twice continuously differentiable on $(G \times G)\backslash\{x=y\}$ and there exists a real number ν ($\nu < n$) such that, for any $x, y \in G$, $x \neq y$, and any multi-indices $\alpha = (\alpha_1, \dots, \alpha_n)$ and $\beta = (\beta_1, \dots, \beta_n)$ with $|\alpha| + |\beta| \leq 2$,

(1.2)

$$\left|\left(\frac{\partial}{\partial x_1}\right)^{\alpha_1} \cdots \left(\frac{\partial}{\partial x_n}\right)^{\alpha_n} \left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial y_1}\right)^{\beta_1} \cdots \left(\frac{\partial}{\partial x_n} + \frac{\partial}{\partial y_n}\right)^{\beta_n} K(x,y)\right| \\ \leq b \begin{cases} 1, & \nu + |\alpha| < 0 \\ 1 + |\log|x-y||, & \nu + |\alpha| = 0\,, \\ |x-y|^{-\nu-|\alpha|}, & \nu + |\alpha| > 0 \end{cases} \quad b = \text{constant}.$$

Here the following usual conventions are adopted:

$$|\alpha| = \alpha_1 + \cdots + \alpha_n \quad \text{for } \alpha = (\alpha_1, \dots, \alpha_n) \in \mathbf{Z}_+^n,$$
$$|x| = (x_1^2 + \cdots + x_n^2)^{1/2} \quad \text{for } x = (x_1, \dots, x_n) \in \mathbf{R}^n.$$

Received by the editors on November 27, 1991.