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NUMERICAL METHODS FOR HYPERBOLIC AND PARABOLIC INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT. An analysis by energy methods is given for fully discrete numerical methods for time-dependent partial integro-differential equations. Stability and error estimates are derived in H^1 and L_2. The methods considered pay attention to the storage needs during time-stepping.

1. Introduction. The main purpose of this paper is to study numerical methods for the solution of the hyperbolic integro-differential equation

$$u_{tt} + A(t)u = \int_0^t B(t,s)u(s)\,ds + f(t), \quad \text{in } \Omega \times J, \tag{1.1a}$$

together with the initial and boundary conditions

$$\begin{gathered} u = 0, \quad \text{on } \partial\Omega \times J, \\ u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \quad \text{in } \Omega, \end{gathered} \tag{1.1b}$$

and for analogous problems for equations of parabolic type. Here Ω is a bounded domain in R^d with smooth boundary $\partial\Omega$, J denotes the interval $[0,T]$ with a fixed upper limit T, $A(t)$ is a self-adjoint, uniformly positive definite uniformly elliptic second order differential operator, and $B(t,s)$ is a second order partial differential operator, both with smooth coefficients. Problems of this nature, and nonlinear versions thereof, occur, e.g., in visco-elasticity, cf. Renardy, Hrusa, and Nohel [**5**] and references therein.

The numerical methods considered in this paper will be obtained by discretizing in space by a Galerkin finite element method, followed by a finite difference and quadrature scheme for the time stepping. They

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