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STABILITY OF COLLOCATION METHODS FOR VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT. We investigate the stability properties of exact and discretized collocation methods, with respect to Volterra integro-differential equations, with degenerate kernel and the basic test equation.

1. Introduction. This paper concerns the stability analysis of the collocation methods for the Volterra integro-differential equation (hereafter referred to as VIDE):

$$y'(t) = f\left(t, y(t), \int_{t_0}^{t} K(t,s,y(s)\,ds\right), \quad t \in [t_0, T], \qquad y(t_0) = y_0 \tag{1.1}$$

where the given functions f and K are assumed to be continuous respectively for $t \in [t_0, T]$ and $(t,s) \in S^* = \{(t,s) : t_0 \le s \le t \le T\}$. For the sake of completeness, the collocation methods and their relationship with the Runge-Kutta methods are described in Section 2.

At present, there are few general analyses on the stability properties of numerical methods for VIDE, and in particular no results are known to the authors about the collocation methods. Until now, the stability analysis has been carried out on the basic test equation (see for example [**3, 5, 6, 11, 15, 16**]) and on positive-definite kernels ([**13, 14**]). In this paper we analyze the stability of the collocation methods, both exact and discretized, with respect to the linear VIDE with degenerate kernel of rank n [**2, 5**]:

$$y'(t) = g(t) + q(t)y(t) + \int_{t_0}^{t} \sum_{l=1}^{n} a_l(t) b_l(s) y(s)\,ds \tag{1.2}$$

where g, q, a_l, b_l; $l = 1, \dots, n$ are assumed to be continuous.

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