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SPECTRAL APPROXIMATIONS FOR WIENER-HOPF OPERATORS II

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ABSTRACT. The comparison of spectral properties of operators

$$Kf(s) = \int_0^\infty \kappa(s-t)f(t)\,dt,$$
$$K_\beta f(s) = \int_0^\beta \kappa(s-t)f(t)\,dt,$$

with $\kappa \in L^1(R)$, which was initiated in [**3**], is extended here in several directions. In [**3**], the operators were defined on the space of bounded continuous functions on the half-line. Now they are studied on $L^2(R^+)$. The spectra are unchanged. Particular attention is paid to the self-adjoint case. There is a very close relationship between spectral properties of K and K_β as $\beta \to \infty$. Under further restrictions, $\sigma(K_\beta)$ is asymptotically dense in $\sigma(K)$ as $\beta \to \infty$. The proofs are based directly on properties of the operators. This enables us to avoid extraneous hypotheses which Fourier transform methods often require.

1. Introduction. In [**3**] we investigated the relationship between the spectrum of a Wiener-Hopf operator

$$Kf(s) = \int_0^\infty \kappa(s-t)f(t)\,dt, \quad s \in R^+ = [0,\infty],$$

and the spectra of the corresponding finite-section operators

$$K_\beta f(s) = \int_0^\beta \kappa(s-t)f(t)\,dt, \quad s \in R^+,\ \beta \in R^+,$$

where $\kappa \in L^1(R)$ and $f \in X^+$, the space of bounded, continuous, real or complex functions on R^+ with $||f|| = \sup|f(t)|$. To avoid trivialities, assume that $||\kappa||_1 \neq 0$. Then $K \neq 0$ and the operator K is not compact. However, the operators K_β are compact.

We proved in [**3**] that every neighborhood of $\sigma(K)$ contains $\sigma(K_\beta)$ for β sufficiently large and that every point in $\sigma(K)$ is an asymptotic