JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 4, Number 3, Summer 1992

THE STRUCTURE OF ALGEBRAS OF SINGULAR INTEGRAL OPERATORS

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1. Introduction. Recently, the application of local principles allowed to get a deep insight into the nature of algebras of singular integral operators with piecewise continuous coefficients. In particular, it turned out that these algebras are isometrically isomorphic to algebras of continuous functions on a Hausdorff compact which take values in certain Banach algebras (see [**4**] and [**10**] for algebras of operators on simple closed curves and [**7**–**9**] for the case of general composed curves).

In this paper we will employ the above-mentioned results to describe the center and the commutator ideal of such algebras, and to give some applications to semi-Fredholm properties of singular integral operators as well as to the decomposition of the algebra of all singular integral operators into simpler objects. The basic results from [**10**] and [**7**–**9**] are concentrated in the first two sections without proofs; for a more comprehensive acquaintance with singular integral operators, we refer to the monographs [**3, 5, 6, 11**].

2. Singular integral operators on the half axis. Given numbers p and α with $p > 1$ and $0 < 1/p + \alpha < 1$, we let $L^p(\alpha)$ refer to the Lebesgue space on the positive half axis $\mathbf{R}^+$ provided with the norm

$$||f|| = \left(\int_0^\infty |f(s)|^p |s|^{\alpha p} \, ds \right)^{1/p},$$

and we define the singular integral operator S on $\mathbf{R}^+$ by

$$(Sf)(t) = \frac{1}{\pi i} \int_0^\infty \frac{f(s)}{s-t} \, ds, \quad t \in \mathbf{R}^+.$$

Under the above restrictions for p and α, the operator S is bounded on $L^p(\alpha)$. Let $\Sigma^p(\alpha)$ stand for the smallest closed subalgebra of the algebra $L(L^p(\alpha))$ of all bounded linear operators on $L^p(\alpha)$ which contains the identity operator I and the singular integral operator S.

Received by the editors on June 8, 1991.