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POSITIVE PERTURBATIONS OF LINEAR VOLTERRA EQUATIONS AND SINE FUNCTIONS OF OPERATORS

ABDELAZIZ RHANDI

Introduction. The purpose of this note is to study perturbations of linear Volterra equations with positive solution families and positive sine functions by positive operators.

Let E be a Banach lattice and A an unbounded closed linear operator in E with dense domain $D(A)$. We say that A is resolvent positive if there exists $w \in \mathbf{R}$ such that $(\mu - A) : D(A) \to E$ is bijective and $(\mu - A)^{-1}$ is a positive operator on E for all $\mu > w$.

Let $a : [0, \infty) \to \mathbf{R}$ be a function which is of bounded variation on each compact interval $[0, T]$, $T > 0$ and consider the linear Volterra equation

$(\mathrm{VO})_A$

$$U(t) := x + a * AU(t) = x + \int_0^t a(t-s)AU(s)\,ds, \quad t \geq 0, \quad x \in D(A).$$

We assume throughout that a is exponentially bounded, i.e., there exist $K \geq 0$, $\beta \geq 0$, such that $|a(t)| \leq K \exp(\beta t)$, $t \geq 0$. Then we can define the function $\mathrm{d}\hat{\mathrm{a}}$ by

$$\mathrm{d}\hat{\mathrm{a}}\,(\mu) = \int_0^\infty \exp(-\mu t)\,\mathrm{da}\,(t), \quad \mu > \beta.$$

We assume further that $\mathrm{d}\hat{\mathrm{a}}\,(\mu) \neq 0$, $\mu > \beta$. A strongly continuous family $(V(t))_{t \geq 0}$ of bounded linear operators on E is called a *solution family* (or a *resolvent*) for $(\mathrm{VO})_A$ if there exist $M \geq 0$, $w \geq \beta$ such that

(i) $||V(t)|| \leq M \exp(wt)$

(ii) $V(0) = 1$

(iii) $(\mu - \mathrm{d}\hat{\mathrm{a}}\,(\mu)A) : D(A) \to E$ is bijective, $\mu > w$ and

$$(\mu - \mathrm{d}\hat{\mathrm{a}}\,(\mu)A)^{-1} = \int_0^\infty \exp(-\mu t)V(t)\,dt.$$

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