

POSITIVE SOLUTIONS OF INTEGRO-DIFFERENTIAL EQUATIONS WITH UNBOUNDED DELAY

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ABSTRACT. We obtain necessary and sufficient conditions for the existence of a solution of the linear integro-differential equation

$$\dot{x}(t) + bx(t) + \int_{-\infty}^t c(t-s)x(s) ds = 0, \quad t \geq 0$$

which is positive for $t > 0$. We also obtain conditions for the oscillation of all solutions of the Volterra-type integro-differential equation of population dynamics

$$\dot{N}(t) = N(t) \left[a - bN(t) - \int_{-\infty}^t c(t-s)N(s) ds \right], \quad t \geq 0.$$

1. Introduction. Our aim in this paper is to obtain necessary and sufficient conditions for the existence of a solution of the linear integro-differential equation

$$(1.1) \quad \dot{x}(t) + bx(t) + \int_{-\infty}^t c(t-s)x(s) ds = 0, \quad t \geq 0$$

which is positive for $t > 0$. We also obtain conditions for the oscillation of all solutions of the Volterra-type integro-differential equation of population dynamics

$$(1.2) \quad \dot{N}(t) = N(t) \left[a - bN(t) - \int_{-\infty}^t c(t-s)N(s) ds \right], \quad t \geq 0.$$

The literature concerning results of the above type is scarce. For some related results, see [3] and [4] and the references cited therein.

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