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# A RECTANGULAR QUADRATURE METHOD FOR LOGARITHMICALLY SINGULAR INTEGRAL EQUATIONS OF THE FIRST KIND

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ABSTRACT. This paper is concerned with a rectangular quadrature method for the numerical solution of a logarithmically singular integral equation of the first kind on a simple closed curve. By extracting the logarithmic singularity, the integral equation is first transformed into an equivalent integral equation with peridic integrands which do not possess singularities. The discretized equation is then obtained by replacing the integrals with a rectangular quadrature rule and by collocating at the quadrature nodes. The resulting system of linear algebraic equations does not involve the evaluation of integrals. The method is analyzed by giving an explicit truncation error formula and a stability proof. As a consequence, the method is proved to have an optimal rate of convergence of $O(h^3)$, where $h$ is the stepsize of the quadrature rule. Based on a derived asymptotic error expansion, Richardson's extrapolation is used to accelerate the convergence up to order $O(h^5)$. Numerical examples are included to illustrate the predicted rates of convergence.

**1. Introduction.** In this paper we consider a rectangular quadrature method for the numerical solution of the singular integral equation of the first kind

$$(1.1) \qquad -\int_\Gamma \log|x-y|\rho(y)\,dl(y) = f(x), \quad x=(x_1,x_2)\in\Gamma,$$

where $\Gamma$ is a simple closed curve in the plane, $dl(y)$ denotes the element of the arc length at a point $y=(y_1,y_2)\in\Gamma$, and $|x-y|$ is the Euclidean distance between $x$ and $y$. The function $f$ is assumed to be given and $\rho$ is the desired solution. Equation (1.1) arises in direct and indirect boundary integral equation methods in the solution of the Dirichlet

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