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WIENER-HOPF-HANKEL OPERATORS FOR SOME WEDGE DIFFRACTION PROBLEMS WITH MIXED BOUNDARY CONDITIONS

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ABSTRACT. An operator theoretic approach is used to study problems of diffraction of time-harmonic electromagnetic (or acoustic) waves by right angle wedges Ω_w such that one of the faces is perfectly conducting (soft) and the other either nonconducting (hard) or imperfectly conducting (with a finite impedance). The correspondent boundary value problems for the two dimensional Helmholtz equation are shown to be well posed in the energy space $H_1(\mathbf{R}^2\backslash\overline{\Omega}_w)$. These problems are reduced to equivalent integral equations in $L_2^+(\mathbf{R})$ of Wiener-Hopf-Hankel type, which can be explicitly solved by obtaining canonical generalized factorizations of certain nonrational 2×2 matrix-valued symbols.

1. Introduction. In this paper we consider the diffraction problem of an electromagnetic (or acoustic) wave by a rectangular wedge $\{(x,y,z)\in\mathbf{R}^3 : x<0, y<0\}$ one of whose faces is perfectly conducting (or soft) and the other face has a prescribed impedance, either finite or infinite. The wedge is supposed to be immersed in a homogeneous and lossy medium, and we assume a time-harmonic incident field with only one component, parallel to the edge $x=y=0$, $z\in\mathbf{R}$ of the wedge.

Splitting the total field into the incident and diffracted field, the above assumptions, together with Maxwell's equations, lead to the following boundary value problem $\mathcal{P}_\lambda$ for the two dimensional Helmholtz equation in the exterior of $\overline{\Omega}_w=\{(x,y)\in\mathbf{R}^2; x\le 0, y\le 0\}$,

$$(\Delta+k_0^2)u(x,y)=0,\qquad (x,y)\in\Omega=\mathbf{R}^2\backslash\overline{\Omega}_w \tag{1.1}$$

$$\left(\frac{\partial}{\partial y}-\lambda\right)u(x,0+)=f(x),\quad x<0 \tag{1.2}$$

$$u(0+,y)=g(y),\quad y<0 \tag{1.3}$$

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