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A QUADRATURE METHOD FOR CAUCHY INTEGRAL EQUATIONS WITH WEAKLY SINGULAR PERTURBATION KERNEL

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ABSTRACT. The authors study the mean weighted convergence of the quadrature method for solving integral equations over the arc $(-1,1)$ with Cauchy kernel and with a perturbation kernel not necessarily regular. Error estimates in uniform norm are also given.

1. Introduction. Many problems in aerodynamics and elasticity lead to a singular integral equation with Cauchy kernel of the form

$$(1.1)\qquad a(x)u(x)+\frac{b(x)}{\pi}\int_{-1}^{1}\frac{u(t)}{t-x}\,dt+\int_{-1}^{1}k(x,t)u(t)\,dt=f(x)$$

on the interval $(-1,1)$ (see, e.g., [**1, 16, 19**]). The first integral in (1.1) is to be interpreted as the Cauchy principal value. Hereby a,b and f are given Hölder continuous functions, and k is a given smooth or weakly singular kernel function.

The problem we are interested in is to find an approximation to the unknown solution u by using projection methods (like collocation or Galerkin schemes) or quadrature procedures with orthogonal polynomials as trial functions. There is already a considerable literature on this subject in the case of regular kernel k (see, e.g., the surveys [**9, 6**–**8, 12, 22, 23, 13**–**15, 24**] and the references given by the same authors). In most of these papers the following strategy is employed. For given functions a and b, one introduces two sets of orthogonal polynomials which are denoted by $\{p_n\}$ and $\{q_n\}$, where $Dp_n=q_{n-\chi}$ with D being the dominant part of Equation (1.1) and χ the index of D (see Section 2). For a given value of n, we use Gauss-type quadrature rules based on the zeros of p_n and collocate at the zeros of $q_{n-\chi}$. In

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