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EXACT SOLUTION OF A SIMPLE HYPERSINGULAR INTEGRAL EQUATION

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ABSTRACT. We obtain the general solution to the simplest one-dimensional hypersingular integral equation; the integral is a Hadamard finite-part integral over a finite interval. We use elementary methods, relating the integral equation to a singular integral equation with a known solution. Despite this, our formula appears to be new.

1. Introduction. We consider the hypersingular integral equation

$$Hf \equiv \frac{1}{\pi} \oint_{-1}^{1} \frac{f(t)}{(x-t)^2}\, dt = v(x), \quad -1 < x < 1. \tag{1.1}$$

Here, $v(x)$ is a known function and $f(x)$ is to be determined. The integral must be interpreted as a Hadamard finite-part integral, defined by

$$\oint_{-1}^{1} \frac{f(t)}{(x-t)^2}\, dt = \lim_{\varepsilon\to 0}\left\{\int_{-1}^{x-\varepsilon} \frac{f(t)}{(x-t)^2}\, dt + \int_{x+\varepsilon}^{1} \frac{f(t)}{(x-t)^2}\, dt - \frac{2f(x)}{\varepsilon}\right\} \tag{1.2}$$

where $|x| < 1$ and f is required to have a Hölder-continuous derivative, $f \in C^{1,\alpha}(-1,1)$. The finite-part integral (1.2) is related to a Cauchy principal-value integral by

$$\oint_{-1}^{1} \frac{f(t)}{(x-t)^2}\, dt = -\frac{d}{dx} \int_{-1}^{1} \frac{f(t)}{x-t}\, dt, \tag{1.3}$$

provided that $f \in C^{1,\alpha}$; indeed, (1.3) is sometimes taken as the *definition* of a finite-part integral. Further properties of finite-part integrals and numerous references to the related literature can be found in [**6**, **7**].

In this short paper, we give the general solution of (1.1) for v in a suitably restricted class of functions. This formula seems to be new, and is obtained by exploiting (1.3).

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