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POSITIVE SOLUTIONS OF A CLASS OF NONLINEAR INTEGRAL EQUATIONS AND APPLICATIONS

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1. Introduction. It is known that many real world problems such as chemical reactors, neutron transport, infectious diseases and etc. can be modeled by nonlinear integral equations [**2,4,5**]. In the study of such problems, we are often interested only in finding positive solutions of a nonlinear integral equation due to the practical meaning of the physical model concerned. In this note we study a class of nonlinear integral equations given by

$$u(x) = \lambda \int_\Omega K(x,y) f(y,u(y))\, dy \tag{1.1}$$

and its nonlinear perturbation

$$u(x) = \lambda \int_\Omega K(x,y) f(y,u(y))\, dy + G(u(x)), \tag{1.2}$$

where $f(x,u)$ is a reciprocal of a polynomial. A prototype of (1.1) is the following integral equation

$$\varphi(x) = \int_0^1 \frac{R(x,y)}{x^2 - y^2} \frac{1}{1+\varphi(y)}\, dy, \tag{1.3}$$

which comes from the integral equation

$$1 = \psi(x) + \psi(x) \int_0^1 \frac{R(x,y)}{x^2-y^2} \psi(y)\, dy \tag{1.4}$$

by a change of variable $\varphi(x) = (1/\psi(x)) - 1$. Equation (1.4) is of interest in nuclear physics [**5**]. This paper is organized as follows. In Section 2 we consider equation (1.1) and prove that for any positive number λ, equation (1.1) has exactly one positive solution which can

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