JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 4, Number 2, Spring 1992

ON THE BEHAVIOR AT INFINITY OF SOLUTIONS OF INTEGRAL EQUATIONS ON THE REAL LINE

S.N. CHANDLER-WILDE

ABSTRACT. We consider integral equations of the form $x(s) = y(s) + \int_{-\infty}^{+\infty} k(s,t)x(t)\,dt$ and the behavior of the solution, x, at infinity. In particular, we show that if, for some $q > 1$, $|k(s,t)| = O(|s-t|^{-q})$, uniformly in s and t, as $|s-t| \to \infty$, some other mild conditions on the kernel k are satisfied, and $y(s) = O(s^{-p})$ as $s \to \infty$, with $0 < p < q$, then $x(s) = O(s^{-p})$. Two examples illustrate the application of these results and the extent to which they may be considered sharp. The second of these examples is a boundary integral equation which models two-dimensional harmonic sound propagation in a half-plane with a variable impedance boundary condition.

1. Introduction. We consider integral equations of the form

$$x(s) = y(s) + \int_{-\infty}^{+\infty} k(s,t)x(t)\,dt, \quad s \in \mathbf{R}, \tag{1}$$

with y and k given and x to be determined. We abbreviate (1) as

$$x = y + Kx \tag{2}$$

where the integral operator K is defined by

$$K\psi(s) = \int_{-\infty}^{+\infty} k(s,t)\psi(t)\,dt. \tag{3}$$

Denote the space of Lebesgue integrable functions on $\mathbf{R}$ by $L_1(\mathbf{R})$ and let $C(\overline{\mathbf{R}})$ denote the Banach space of bounded continuous functions on $\mathbf{R}$. Let $k_s(t) = k(s,t)$. Throughout the paper, we assume that $k_s \in L_1(\mathbf{R})$ for $s \in \mathbf{R}$ and that k satisfies the following hypotheses:

A. $\sup_{s\in\mathbf{R}} ||k_s||_1 = \sup_{s\in\mathbf{R}} \int_{-\infty}^{+\infty} |k(s,t)|\,dt < \infty$.

Received by the editors on June 22, 1991.