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AN UNCONVENTIONAL QUADRATURE METHOD FOR LOGARITHMIC-KERNEL INTEGRAL EQUATIONS ON CLOSED CURVES

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ABSTRACT. A new, fully discrete method is proposed for the logarithmic-kernel integral equation of the first kind on a smooth closed curve. The method uses two levels of numerical quadrature: a trapezoidal rule for the integral containing the logarithmic singularity; and a special quadrature rule for the outer integral, which compensates, in part, for the errors in the first integral. A convergence and stability analysis is given, and the predicted orders of convergence verified in a numerical example. A numerical experiment suggests that the method can be useful even for a curve with corners.

1. Introduction. In this paper we propose and analyze a fully discrete method for the approximate solution of

$$-\frac{1}{\pi}\int_\Gamma \log|t-s|z(s)\,dl_s = g(t), \quad t\in\Gamma, \tag{1.1}$$

where z is an unknown function, dl_s the element of arc-length, $|t-s|$ the Euclidean distance between $t,s\in\Gamma$, and Γ a smooth simple closed curve in the plane. The curve is assumed to have transfinite diameter (or conformal radius) different from 1, in which case (1.1) has a unique solution.

If we assume that Γ can be parametrized by a 1-periodic C^∞ function $\nu:\mathbf{R}\to\Gamma$, with $|\nu'(x)|\neq 0$, then (1.1) can be written

$$-\int_0^1 2\log|\nu(x)-\nu(y)|u(y)\,dy = f(x), \quad x\in[0,1], \tag{1.2}$$

or

$$Lu = f, \tag{1.3}$$

where

$$u(x) = z(\nu(x))|\nu'(x)|/(2\pi), \quad x\in[0,1], \tag{1.4}$$