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THE NUMERICAL APPROXIMATION OF THE SOLUTION OF A NONLINEAR BOUNDARY INTEGRAL EQUATION WITH THE COLLOCATION METHOD

M. HAMINA, K. RUOTSALAINEN AND J. SARANEN

ABSTRACT. Recently, Galerkin and collocation methods have been analyzed in connection with the nonlinear boundary integral equation which arises in solving the potential problem with a nonlinear boundary condition. Considering this model equation, we propose here a discretized scheme such that the nonlinearity is replaced by its L^2-orthogonal projection. We are able to show that this approximate collocation scheme preserves the theoretical L^2-convergence. For piecewise linear continuous splines, our numerical experiments confirm the theoretical quadratic L^2-convergence.

1. Introduction. We consider the solution of the potential equation in a bounded domain Ω with a given Neumann-type nonlinear boundary condition. Taking the model problem of [**12, 13**], consider

$$\begin{cases} \Delta\Phi = 0, & \text{in } \Omega \\ -\partial_n \Phi|_\Gamma = f(x,\Phi) - g, & \text{on } \Gamma = \partial\Omega. \end{cases} \tag{1.1}$$

We assume that the boundary Γ is a smooth Jordan-curve in the plane. Conditions for the nonlinear function $f(x,\Phi)$ as well as for the given boundary data g will be specified later.

By using Green's representation formula for the potential Φ, problem (1.1) reduces to the following nonlinear boundary integral equation [**13**]

$$\frac{1}{2}u - Ku + VF(u) = Vg. \tag{1.2}$$

Here V is the single layer boundary integral operator

$$Vu(x) := \frac{-1}{2\pi}\int_\Gamma u(y)\ln|x-y|\,ds_y,$$

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