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INTEGRABLE SOLUTIONS OF A FUNCTIONAL-INTEGRAL EQUATION

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ABSTRACT. We consider a very general functional-integral equation and we prove the existence of integrable solutions of this equation.

In this paper we consider the following functional-integral equation

$$y(t) = f\left(t, r\int_0^1 k(t,s)g(s,y(s))\,ds\right) \quad t \in [0,1] \tag{1}$$

and we prove that, under very general hypotheses, it admits a solution $x \in L^1[0,1]$. We observe that if $f(t,u) = \varphi(t) + u$ we get Hammerstein integral equations (we refer to [**2, 5, 9**] and references therein for papers about existence results concerning this equation as well as for applications of it to other questions), whereas when $g(s,v) = v$ we obtain a functional-integral equation recently studied in [**3**], where the usefulness of it in applications was also pointed out. Our theorem extends all of the known results from [**2, 3, 5, 6, 7** and **9**] because the hypotheses we consider are very general and *natural* in the sense that they are necessary and sufficient conditions for certain (superposition) operators to take $L^1[0,1]$ into itself continuously, see [**8**].

We remark that in the results from [**2, 3, 5**, and **9**] assumptions of monotonicity and coercivity were quite often assumed by the authors, whereas we dispense completely with them; furthermore, in [**3**] Banas and Knap assumed that $k(t,s) \geq 0$ a.e. on $[0,1]^2$; we are able to dispense with this requirement as well as with the following other hypothesis:

There exists $\lambda \in L^1[0,1]$ *such that* $|k(t,s)| \leq \lambda(t)$
t *a.e. on* $[0,1]$, $s \in [0,1]$

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