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THE METHOD OF LINES FOR PARABOLIC PARTIAL INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT. We present a method of lines approximation of the solution of a particular nonlinear Volterra partial integro-differential equation. Discretization in space of this equation leads to a system of stiff integro-differential equations. In a second step, this system is integrated in time by the implicit Euler method.

The concept of the logarithmic norm, introduced in the theory of numerical methods for stiff ordinary differential equations, plays an important role in the convergence analysis.

1. Introduction. We consider the nonlinear, parabolic-type Volterra partial integro-differential equation (VPIDE)

(1.1)
$$u_t(x,t) = g(x,t) + \sum_{i=0}^{2} a_i(x,t)\frac{\partial^i u}{\partial x^i}(x,t) + \int_0^t b(x,t,s,u(x,s))\,ds, \qquad 0 < x < 1, \quad 0 < t \le T,$$

where $u_t := \partial u/\partial t$. The functions g, a_i, $i = 0,1,2$, and b are continuous on $D := \{(x,t) \in \mathbf{R}^2 : x \in I, t \in J\}$ and $I \times S \times \mathbf{R}$, respectively, with $I := [0,1]$, $J := [0,T]$ and $S := \{(t,s) \in J \times J : s \le t\}$. We assume that the function b is continuously differentiable and therefore satisfies a Lipschitz condition with respect to its last variable, i.e., there exists a constant $L \ge 0$ such that for all $u, v \in \mathbf{R}$, $x \in I$ and $(t,s) \in S$,

$$|b(x,t,s,u) - b(x,t,s,v)| \le L|u-v|. \tag{1.2}$$

We exclude the case $a_1 = a_2 \equiv 0$. To equation (1.1) we associate the following initial and boundary conditions:

$$u(x,0) = \phi(x), \quad x \in I, \tag{1.3}$$
$$u(0,t) = u(1,t) = 0, \quad t \in J, \tag{1.4}$$

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