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A SURVEY OF NUMERICAL METHODS FOR SOLVING NONLINEAR INTEGRAL EQUATIONS

KENDALL E. ATKINSON

ABSTRACT. A survey is given of numerical methods for calculating fixed points of nonlinear integral operators. The emphasis is on general methods, ones that are applicable to a wide variety of nonlinear integral equations. These methods include projection methods (Galerkin and collocation) and Nyström methods. Some of the practical problems related to the implementation of these methods is also discussed. All of the methods considered require the solution of finite systems of nonlinear equations. A discussion is given of some recent work on iteration methods for solving these nonlinear equations.

1. Introduction. In the following survey, we consider numerical methods of a general nature, those that can be applied to a wide variety of nonlinear integral equations. The integral equations are restricted to be of the second kind,

$$x = \mathcal{K}(x) \tag{1.1}$$

where $\mathcal{K}$ is a nonlinear integral operator. Important special cases include Hammerstein and Urysohn integral operators.

The Hammerstein integral equation is

$$x(t) = y(t) + \int_D K(t,s) f(s, x(s))\, ds, \quad t \in D. \tag{1.2}$$

with D a closed region or manifold in $\mathbf{R}^m$, some $m \geq 1$. A well-known example is the *Chandrasekhar H-equation*

$$H(t) = 1 + \frac{c}{2} \int_0^1 \frac{tH(t)H(s)}{t+s}\, ds \tag{1.3}$$

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