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NONLINEAR INTEGRAL EQUATIONS ON THE HALF LINE

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Dedicated to Professor John A. Nohel in appreciation
for his many important contributions to the study
of integral equations.

ABSTRACT. This paper treats the existence and approximation of solutions of nonlinear integral equations defined on the half line $[0, \infty)$. Integral equations on $[0, \infty)$ are approximated by finite-section approximations, which reduce to integral equations on bounded intervals $[0, \beta]$. In the case when solutions are unique, the solutions x_β to the finite-section approximations converge uniformly on compact sets to the solution x of the integral equation on $[0, \infty)$, under natural hypotheses on its kernel. When solutions are not unique, the solution sets of the finite-section approximations converge in an appropriate sense to the solution set of the given integral equation. Integral equations of the type treated here include certain nonlinear Wiener-Hopf equations and integral equations of Hammerstein type. There are implications pertaining to global existence questions for nonlinear initial and boundary value problems for ordinary differential equations, in particular for a semi-conductor problem.

1. Introduction. We shall consider nonlinear integral equations on the half line $R^+ = [0, \infty)$ of the form

$$x(s) - \int_0^\infty k(s, t, x(t))\, dt = y(s), \tag{1.1}$$

and more general nonlinear operator equations. By hypothesis, x and y are bounded, continuous functions on R^+. Assumptions on k will be imposed later.

Finite-section approximations for (1.1) are given by

$$x_\beta(s) - \int_0^\beta k(s, t, x_\beta(t))\, dt = y(s), \tag{1.2}$$

for $\beta \geq 0$. Since (1.2) determines $x_\beta(s)$ for $s > \beta$ in terms of $x_\beta(t)$ for $t \in [0, \beta]$, (1.2) reduces to an integral equation on $[0, \beta]$.