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MESH INDEPENDENCE OF NEWTON-LIKE METHODS FOR INFINITE DIMENSIONAL PROBLEMS

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ABSTRACT. Globally convergent modifications of Newton's method, such as the Armijo rule, can be applied to infinite dimensional problems and their discretizations. We show that if the construction of the discretizations is done properly, then the convergence behavior of the iteration is the same for the discrete problems as it is for the infinite-dimensional problem. Basic to these results is the use of the concept of discrete convergence as a tool to measure the performance of algorithms and a new setting of Banach spaces with incomplete metrics, for example, norms generated by continuous inner products. The motivating problems are integral equations with continuous kernels. This result extends to the globally convergent case results of Allgower, Böhmer, Potra, and Rheinboldt, and the authors. In addition, we strengthen the previous results on mesh independence of quasi-Newton methods. Numerical results are reported that illustrate the results.

1. Introduction. Many methods for solution of nonlinear equations in $\mathbf{R}^n$ depend on inner product information. Quasi-Newton methods, such as Broyden's method [**6**] or the BFGS method [**7, 10, 11, 27**] use inner products to construct approximations to Jacobian and Hessian matrices. Globally convergent modifications of Newton's method, such as line searches or trust region strategies, use inner product norms to test for sufficient decrease and for computation of gradients and steepest descent directions. When such methods are applied to discretizations of equations in Banach spaces these inner products impose a Hilbert space structure on the discretized problem that may not be appropriate for the infinite dimensional problem. Other, more general, global convergence methods may use merit functions that satisfy crucial

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