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ON A FORCED QUASILINEAR HYPERBOLIC VOLTERRA EQUATION WITH FADING MEMORY

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ABSTRACT. In this paper we prove the global existence of a solution to a boundary initial value problem for a forced quasilinear hyperbolic Volterra equation under the assumption that the forcing term remains small and can be decomposed into a time-periodic part and a part that decays to zero as $t \to \infty$. We also show that the solution converges to a time-periodic function as $t \to \infty$; the latter is a periodic solution of a related history value problem.

1. Introduction. In this paper we consider global existence and asymptotic behavior of solutions of the problem

$$
(1.1)\qquad \begin{aligned}
u_t &= \int_0^t a(t-\tau)\sigma(u_x)_x \, d\tau + f(t,x), && \text{for } x \in (0,1),\ t > 0,\\
u(0,x) &= u_0(x), && \text{for } x \in (0,1),\\
u(t,0) &= u(t,1) = 0, && \text{for } t \geq 0.
\end{aligned}
$$

Here $a : (0,\infty) \to R$, $\sigma : R \to R$ is a given smooth function, the data $f : (0,\infty) \times (0,1) \to R$ and $u_0 : (0,1) \to R$ are sufficiently smooth functions compatible with the boundary conditions.

The initial boundary value problem (1.1) has been studied by many authors. In [**7**] MacCamy established a global existence result for the problem (1.1) and showed that the problem (1.1) is related to a theory of heat flow in materials with memory. The existence of global solutions for (1.1) was also established by Dafermos and Nohel [**1**] and Staffans [**11**]. These global existence results treat the case that the initial datum u_0 is sufficiently small and the forcing term f is sufficiently small and decays to 0 as $t \to \infty$.

The purpose of this paper is to study the global existence and asymptotic behavior of solutions for (1.1) in the case that the forcing term f remains small but does not necessarily decay to zero as t tends to ∞. More precisely, we treat the case that f is sufficiently small and can be written in the form $f_1 + f_2$, where f_1 is a time periodic function