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MULTI-STEP METHODS FOR FIRST KIND SINGULAR VOLTERRA INTEGRAL EQUATIONS

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ABSTRACT. Multi-step methods are described for first kind singular Volterra integral equations. Methods of order three are constructed and illustrated with a numerical example.

1. Introduction. Consider the linear Volterra integral equation with weakly singular kernel of the form

$$by(x) + \int_0^x (x-t)^{-\alpha} K(x,t)y(t)\,dt = f(x), \qquad 0 \le t \le x \le a,\ 0 \le \alpha < 1 \tag{1.1}$$

where K and f are given functions and y is the unknown function to be found. The constant b equals 1 or 0 corresponding to a second or first kind equation, respectively. Conditions on K and f which provide for a unique solution $y(x) \in C[0,a]$ may be found in Pogorzelski [**17**], for example.

The methods considered in Section two are applicable to all equations of the form (1.1). This is a nontrivial problem for the case $b = 0$ where it has been well known for almost 20 years that a root condition must be imposed on the weights of the quadrature rule method along with the usual consistency conditions, for $\alpha = 0$ even. Methods for the case $b = 1$ may also be found in Brunner [**2**], Garey [**6, 7**], DeHoog and Weiss [**11**], and Linz [**12**]. Single step methods for the case $b = 0$ may be found in Brunner [**1**], Linz [**13**], and Weiss and Anderssen [**19**].

Cameron and McKee [**3**] have considered multi-step methods for the case $b = 0$. However, their root condition is more intractable than the one given here since they require knowledge about the roots of a power series. Lubich [**14**] has recently constructed methods which are based on having a multi-step method (ρ, σ) of order p with both the

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