JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 3, Number 4, Fall 1991

ON IMPLICITLY LINEAR AND ITERATED COLLOCATION METHODS FOR HAMMERSTEIN INTEGRAL EQUATIONS

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ABSTRACT. Recently, Kumar and Sloan introduced and analyzed a new collocation-type method (which in the following will be referred to as implicitly linear collocation) for the numerical solution of Hammerstein integral equations. In the present paper we discuss the connection between implicitly linear collocation and iterated spline collocation. The results are then extended to a class of nonlinear Volterra-Fredholm integral equations.

1. The implicitly linear collocation method. Spline collocation methods and their iterated and discretized variants for linear Fredholm integral equations of the second kind have been studied extensively during the last 15 years (compare, for example, the survey paper [**4**, pp. 569–578, 584–588]). More recently, much of this analysis has been extended to nonlinear Fredholm equations, either to general Urysohn equations or to Hammerstein equations (see [**1**] for a comprehensive description of the state of the art; compare also [**2**]). In the case of nonlinear Fredholm integral equations of Hammerstein type,

$$y(t) = g(t) + \int_0^T k(t,s)G(s,y(s))\,ds, \quad t \in I := [0,T]. \tag{1.1}$$

Kumar and Sloan [**12**] (see also [**10, 11, 8**]) suggested a new collocation-type method (which, for reasons that will be given in a moment, we will refer to as *implicitly linear collocation*). Setting $z(t) := G(t, y(t))$, the above Hammerstein integral equation (1.1) can be written in "implicitly linear" form,

$$z(t) = G\left(t, g(t) + \int_0^T k(t,s)z(s)\,ds\right), \quad t \in I; \tag{1.2a}$$

This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC Grant A9406).

Mathematics Subject Classification (1980): 65R20, 45G10, 45L10.