JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 3, Number 3, Summer 1991

A KAC-FEYNMAN INTEGRAL EQUATION FOR CONDITIONAL WIENER INTEGRALS

CHULL PARK AND DAVID SKOUG

ABSTRACT. Let $F(x) = \exp\{\int_0^t \theta(s, \int_0^s h(u)\,dx(u))\,ds\}$, for x an element of Wiener space $C[0,T]$ and potential function $\theta(\cdot,\cdot) : [0,T] \times \mathbf{R} \to \mathbf{C}$. In this paper we show that the conditional Wiener integral, $E(F|X)$, with conditioning function $X(x) = \int_0^t h(u)\,dx(u)$, satisfies the Kac-Feynman integral equation. We also consider vector-valued conditioning functions $X(x)$, as well as potentials $\theta(s,\cdot)$ that are Fourier-Stieltjes transforms of Borel measures on $\mathbf{R}$.

1. Introduction. Let $(C[0,T], \mathcal{F}, m_w)$ denote Wiener space where $C[0,T]$ is the space of all continuous functions x on $[0,T]$ with $x(0) = 0$. Let $F(x)$ be a Wiener integrable function on $C[0,T]$, and let $X(x)$ be a Wiener measurable function on $C(0,T]$. In [**10**], Yeh introduced the concept of the conditional Wiener integral of F given X, $E(F|X)$, and for the case $X(x) = x(T)$ obtained some very useful results including a Kac-Feynman integral equation. Further work involving conditional Wiener integrals include [**3, 4, 8,** and **11**]. In [**9**], Park and Skoug extended the theory to include very general conditioning functions, including conditioning functions of the form $X(x) = (\int_0^T \alpha_1(s)\,dx(s), \dots, \int_0^T \alpha_n(s)\,dx(s))$.

A very important class of functions in quantum mechanics are functions on $C[0,T]$ of the form

$$G(x) = \exp\left\{\int_0^T \theta(s, x(s))\,ds\right\}$$

where $\theta : [0,T] \times \mathbf{R} \to \mathbf{C}$. These functions are clearly contained in the class of functions of the form

(1.1)
$$F(x) = \exp\left\{\int_0^T \theta\left(s, \int_0^s h(u)\,dx(u)\right) ds\right\}, \quad h \in L_2[0,T],\ h \neq 0 \text{ a.e.}$$