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NONTRIVIAL SOLUTIONS FOR A CLASS OF NONLINEAR VOLTERRA EQUATIONS WITH CONVOLUTION KERNEL

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ABSTRACT. We consider the Volterra integral equation

$$u(x) = \int_0^s k(x-s)g(u(s))\,ds, \quad x \geq 0,$$

where $k \geq 0$ is an integrable function and g is an increasing absolutely continuous function ($g(0) = 0$) which does not satisfy a Lipschitz condition. New necessary and sufficient conditions for the existence of positive nontrivial solutions are obtained.

1. Introduction. The nonlinear Volterra integral equation with convolution kernel

$$u(x) = \int_0^x k(x-s)g(u(s))\,ds, \quad x \geq 0 \tag{1.1}$$

has been studied recently in the modeling of problems in nonlinear diffusion and shock-wave propagation [**4, 7**]. In these problems the kernel function is nonnegative and g is an increasing continuous function such that $g(0) = 0$. Moreover, g does not satisfy a Lipschitz condition in the vicinity of the origin. A typical example of such a function g is $g(u) = u^p$, $p \in (0,1)$. Obviously, $u \equiv 0$ is the trivial solution to (1.1). But the question of physical interest is the existence of nontrivial solutions to (1.1), i.e., continuous functions u such that $u(x) > 0$ for $x > 0$.

Some particular answers concerning the existence of nontrivial solutions can be found in Gripenberg's work [**3**]. Under restrictive assumptions concerning g, the author has presented there a condition which is necessary and sufficient for the existence of a nontrivial solution to (1.1)

Key words and phrases. Nonlinear Volterra equation, existence of nontrivial solutions, positive solutions.
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