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# REMARKS ON THE EXISTENCE AND UNIQUENESS OF SOLUTIONS OF VOLTERRA FUNCTIONAL EQUATIONS IN $L^p$ SPACES

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**1.** Existence and uniqueness problems for the Volterra integral equations were discussed by many authors. Usually the solutions were sought in the space of continuous functions.

In the literature on the subject there is not much more than the classical result for integral equations in $L^p$ or $L^2$ spaces which one can find in [**5, 10**]. Similar results for multidimensional integral equations in $L^2$ spaces appeared in [**2**]. Recently, the author of the present paper has shown [**8, 9**] that Bielecki's technique of weighted norms [**3**] (which was successfully employed by many authors dealing with integral equations in the space of continuous functions—see [**6**] and a review paper [**4**]) can be applied fairly well to integral equations in $L^p$ spaces.

The aim of the present note is to show how the comparison method works in the case of $L^p_{\text{loc}}$ spaces (for an abstract formulation of the method consult the paper [**7**]) and to discuss different approaches to the problem in the case of a linear comparison function $\Omega$ (see Assumption A).

**2.** Let $a, b \in \mathbf{R}^n$, $a = (a_1, \dots, a_n)$, $b = (b_1, \dots, b_n)$; we write $a \leq b$ if $a_i \leq b_i$, $i = 1, 2, \dots, n$. Put

$$I = [a, b) = \{t : t \in \mathbf{R}^n,\ a \leq t < b\}.$$

The case when $b = (+\infty, \dots, +\infty)$ is accepted. We call this set an interval. Let $B$ be a Banach space with a norm $|\cdot|$. The symbol $L^p_{\text{loc}}(I, B)$ will denote the space of all locally Bochner integrable functions $x$ (Bochner integrable on every compact subset $I_c$ of $I$) for

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