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FORCED VIBRATIONS IN ONE-DIMENSIONAL NONLINEAR VISCOELASTICITY

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ABSTRACT. We prove the existence of time-periodic weak solutions to the integro-differential equation

$$U_{tt}(x,t) = f(U_x(x,t))_x + \int_0^\infty \dot{a}(s) f(U_x(x,t-s))_x \, ds + g(x,t),$$

where g is a time-periodic function.

The main idea of the proof is to construct invariant regions for a parabolic system arising as a viscosity regularization of the original problem. Consequently, we are able both to find a sequence of approximate (viscosity) time-periodic solutions via the Schauder fixed point technique and to pass to the limit using the ideas of compensated compactness.

1. Introduction and statement of results. We consider the motion of a one-dimensional body (string or bar) with undistorted reference configuration $J = (0,l)$, a connected open subset of $\mathbf{R}^1$. Denoting by $U = U(x,t)$ the displacement at the instant t of the point with reference position $x \in J$, we make the following assumption relating the stress $\mathcal{G}$ to the motion of the form

$$\text{(C)} \qquad \mathcal{G}(x,t) = f(U_x(x,t)) + \int_0^\infty \dot{a}(s) f(U_x(x,t-s)) \, ds,$$

where a dot indicates differentiation with respect to time.

Assumptions concerning the kernel a are motivated by a simple model for a material with fading memory (see Hrusa, Nohel, Renardy [**13**] and also Renardy, Hrusa, Nohel [**21**] for an excellent survey):

(A1) $a : [0,\infty) \to [0,\infty)$ is smooth, $a(0) < 1$, a, da/dt, $d^2a/dt^2 \in L_1(0,\infty)$, a is strongly positive (see Nohel, Shea [**19**]),

(A2) $d^k a/dt^k \in L_1(0,\infty)$, $k = 0, \dots, 4$.

We assume, for simplicity, that the body is homogeneous with unit density. Under these circumstances the motion is governed by the equation

$$\text{(E)} \qquad U_{tt}(x,t) = \mathcal{G}_x(x,t) + g(x,t), \quad x \in J, \ t \in \mathbf{R}^1,$$