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PROBABILISTIC ANALYSIS OF NUMERICAL METHODS FOR INTEGRAL EQUATIONS

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ABSTRACT. The approximate solution of Fredholm integral equations is analyzed from a probabilistic point of view. With Wiener type measures on the set of kernels and right-hand sides we determine statistical features of the approximation process—the most likely rate of convergence and the dominating individual behavior. The analysis is carried out for two typical algorithms—the Galerkin and the iterated Galerkin method.

Introduction. The aim of the probabilistic analysis is best explained in a concrete example. Therefore, we first describe the numerical problem and the algorithms to be studied. We consider the Fredholm integral equation

$$x(s) - \int_0^{2\pi} k(s,t)x(t)\,dt = y(s),$$

where $y \in L_2(\Gamma)$, $k \in L_2(\Gamma^2)$, and Γ is the unit circle. Let us write this equation in the form

$$x - T_k x = y$$

and assume that $I - T_k$ (I the identity) is invertible. The Galerkin method seeks an approximate solution $x_n^G \in X_n$ satisfying

$$(x_n^G - T_k x_n^G, z) = (y, z)$$

for all $z \in X_n$, where we let X_n be the space of trigonometric polynomials of degree at most n, and $(\ ,\ )$ denotes the scalar product in $L_2(\Gamma)$. A second algorithm, the iterated Galerkin method (see, e.g., [**17**]), uses x_n^G to obtain a further approximation x_n^I with

$$x_n^I - T_k x_n^G = y.$$

The error analysis is usually based on smoothness assumptions. Let $r \geq s \geq 0$ and let $BH^r(\Gamma^2)$ and $BH^s(\Gamma)$ be the unit balls of the