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AN APPLICATION OF FINITE HILBERT TRANSFORMS IN THE DERIVATION OF A STATE SPACE MODEL FOR AN AEROELASTIC SYSTEM

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ABSTRACT. Dynamic modeling of various aeroelastic control systems requires, at some point in the derivation of the model, an application of Söhngen's inversion formula for finite Hilbert transforms to obtain a desired representation for the solution of the airfoil equation. Conditions on initial data to guarantee well-posedness of the resulting model equations must be matched with those needed to justify the validity of the inversion formula. We show that this compatibility can be achieved by assuming that the circulation history belongs to a weighted L_2 space. The resulting state space formulation provides a suitable setting for control design for the aeroelastic system.

1. Introduction. In recent years the feasibility and advantages of active control surfaces to reduce maneuver, gust and fatigue loads and dampen vibration that contributes to flutter have been extensively studied [**1, 2, 21, 25**]. A systematic procedure for control design requires the development of a "realistic" mathematical model that predicts the dynamic behavior of the physical system. The development of state space models for aeroelastic systems, including unsteady aerodynamics, is potentially important for the design and development of highly maneuverable aircraft.

In [**8**] a complete dynamic model was formulated in terms of a functional differential equation of neutral-type for the elastic motions of a three-degree-of-freedom "typical" airfoil section, with flap, in a two-dimensional, incompressible flow (Theodorsen's problem). In subsequent papers [**10, 11, 12, 13, 24**] the well-posedness of the

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