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GENERIC HOPF BIFURCATION IN A CLASS OF INTEGRO-DIFFERENTIAL EQUATIONS

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Dedicated to John Nohel in commemoration of his 65^{th} birthday.

ABSTRACT. The Hopf bifurcation structure for a class of scalar functional differential equations is examined. It is shown that, within some classes of nonlinear perturbations, points of potentially nongeneric bifurcation can be identified by the orientation of the neutral stability curve associated with the linearized problem. A general "normal form" equation is derived which effectively determines the behavior of generic bifurcations. Practical computational issues are addressed and illustrated with a specific application first considered by Levin and Nohel [**6**].

I. Introduction. Consider the linear scalar equation

$$\dot{x}(t) = \alpha x + \beta \int_{-\infty}^{0} x(t+s)\, d\eta(s) \tag{1.1}$$

and the nonlinear perturbation

(1.2)

$$\begin{aligned}\dot{x}(t) = \alpha x &+ \beta \int_{-\infty}^{0} x(t+s)\, d\eta(s)\\ &+ a_2 x^2(t) + a_2 x^3(t) + b_2 x(t) \beta \int_{-\infty}^{0} x(t+s)\, d\eta(s)\\ &+ b_3 x^2(t) \beta \int_{-\infty}^{0} x(t+s)\, d\eta(s) + c_2 \beta \int_{-\infty}^{0} x^2(t+s)\, d\eta(s)\\ &+ c_3 x(t) \beta \int_{-\infty}^{0} x^2(t+s)\, d\eta(s) + d_3 \beta \int_{-\infty}^{0} x^3(t+s)\, d\eta(s) \dots ,\end{aligned}$$

where the measure $d\eta$ is a suitably normalized positive measure. It is well known that, under certain natural assumptions about the measure $d\eta$, the asymptotic behavior of solutions to (1.1) can be determined by