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MAXIMAL REGULARITY OF LINEAR VECTOR-VALUED PARABOLIC VOLTERRA EQUATIONS

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In Honor of Professor John A. Nohel
on the Occasion of his 65th Birthday

ABSTRACT. Maximal regularity in C^α-spaces of linear Volterra equations in a Banach space X of the form

$$(*)\qquad u(t) = f(t) + \int_0^t a(t-\tau)Au(\tau)\,d\tau, \quad t \geq 0,$$

is studied. The conditions which ensure maximal regularity involve a parabolicity condition for $(*)$, but also some regularity conditions on the kernel $a(t)$. As an illustration of the results, examples from the theory of viscoelasticity and heat conduction in materials with memory are discussed.

1. Introduction. Let X be a Banach space, A a closed linear operator in X with dense domain $D(A)$, $a \in L^1_{\text{loc}}(\mathbf{R}_+)$, and $f \in C(J;X)$, where $J = [0,T]$. We consider the following vector-valued Volterra equation of scalar type:

$$(1)\qquad u(t) = f(t) + \int_0^t a(t-\tau)Au(\tau)\,d\tau, \quad t \in J.$$

In the sequel $*$ will be used for the convolution of two functions on the halfline. Recall that $u \in C(J;X)$ is called a *mild solution* of (1) if $a * u \in C(J;X_A)$ and

$$(2)\qquad u(t) = f(t) + A(a*u)(t), \quad t \in J,$$

holds; here X_A denotes the Banach space $D(A)$ equipped with the graph norm of A. A *strong solution* of (1) is a function $u \in C(J;X_A)$ such that (1) holds on J.

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