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A SLEWING BEAM PROBLEM

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ABSTRACT. We consider feedback stabilization of a simple mechanical system involving a flexible viscoelastic beam. Though we use linear constitutive equations, nonlinearities come in by asymmetry of the geometric configuration of the system. The nonlinearities are handled as Lipschitz perturbations of a linear C_0-semigroup. Using Lyapunov techniques, we prove that solutions of the purely elastic stabilized system converge to zero weakly; if viscoelastic damping is present, the convergence is exponential in energy norm.

1. Introduction. In recent papers we have considered the model of a satellite set up in [**8**]. The system consists of a rigid hub with four identical flexible radial beams rigidly attached. The system is free to rotate and translate in a plane orthogonal to the axis of the hub. The beams are described by the Euler-Bernoulli model, based on a linearly viscoelastic constitutive equation for their material. A control torque can be applied on the hub in order to control its angular position. If we assume that all beams vibrate in phase, the model equations add up to a linear system of the type treated in [**3**] and [**4**]. In these papers it is proved that linear feedback from angular position to torque can force the potential energy stored in deflection and the part of kinetic energy contained in rotation to decay to zero exponentially.

In the model described above the deformation of the flexible parts has no bearing on the location of the center of gravity, as the beams are arranged symmetrically around the axis, and their vibrations are assumed to be synchronized. The latter hypothesis is, of course, somewhat unrealistic. The former assumption imposes a severe restriction on the system's generality. We consider, therefore, the case of a mechanical system, where this symmetry is broken. The simplest one, and

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