JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 3, Number 1, Fall 1991

SOME EXISTENCE RESULTS FOR A NONLINEAR HYPERBOLIC INTEGRODIFFERENTIAL EQUATION WITH SINGULAR KERNEL

STIG-OLOF LONDEN

ABSTRACT. We consider the nonlinear Volterra integro-differential equation

$$u_t(t,x) - \int_0^t a(t-s)\sigma(u_x(s,x))_x\,ds = f(t,x), \quad t \geq 0,\ x \in \mathbf{R},$$

with initial function $u(0,x) = u_0(x)$. We prove existence of global (in time) smooth solutions in the case where the data are small, assuming only $a' \in L^1(\mathbf{R}^+)$ and strong positivity on the kernel. A local existence result for large data is obtained. The proofs use approximating kernels, uniformly of strong positive type and energy estimates.

1. Introduction. The equation

$$\text{(V)} \qquad \begin{gathered} u_t(t,x) - \int_0^t a(t-s)\sigma(u_x(s,x))_x\,ds = f(t,x), \quad t \geq 0,\ x \in \mathbf{R}, \\ u(0,x) = u_0(x), \end{gathered}$$

where $a(t)$ is positive in some sense, presents a bridge between problems of a nonlinear parabolic and problems of a nonlinear hyperbolic nature. If $a(t) \equiv 1$, then (V) is nonlinear hyperbolic; if $a(t)dt$ is a pure point mass at the origin, then (V) is nonlinear parabolic. In the intermediate case, where $a(t)$ is positive and, say, decreasing, convex and in some sense singular at the origin, one may expect solutions combining features of both the extreme cases.

In the linear case, where $\sigma(u) = ku$, this has been established in much detail. Roughly speaking, the more singular the kernel is at the origin, the more smoothing out of initial conditions does the solution present. In fact, the finite propagation speed of the wave equation and the smoothing properties of the heat equation may coexist. See [**3, 7**] and the references mentioned therein, [**8, 15**], and [**16**].