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VISCOELASTIC AND BOUNDARY FEEDBACK DAMPING: PRECISE ENERGY DECAY RATES WHEN CREEP MODES ARE DOMINANT

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Dedicated to John Nohel on the occasion of his sixty-fifth birthday

ABSTRACT. For a linear Volterra equation of scalar type in a Banach space, sufficient conditions are given for the operator norms of three associated resolvent kernels to be integrable with respect to a weight on the positive half-line. The results and methods extend those introduced by Prüss for integrability with respect to ordinary Lebesgue measure. The estimates are applied to the estimation of precise decay rates for energy in a viscoelastic solid when the memory kernel decays algebraically and creep modes dominate the oscillating modes. It is shown that boundary feedback is ineffective in promoting decay in such cases.

1. Introduction. We give sufficient conditions for three resolvent kernels associated with the problem

$$\ddot{\mathbf{u}}(t) = E\mathbf{L}\mathbf{u}(t) + \frac{d}{dt}\int_0^t a(t-\tau)\mathbf{L}\mathbf{u}(\tau)\,d\tau \quad \left(\dot{} = \frac{d}{dt}\right), \qquad \mathbf{u}(0) = \mathbf{u}_0, \quad \dot{\mathbf{u}}(0) = \mathbf{u}_1 \tag{1.1}$$

to be integrable with respect to certain weight functions on $\mathbb{R}^+ \equiv [0,\infty)$. Here $E > 0$, $\mathbf{L}$ is the generator of a strongly continuous cosine family in a Banach Space $\mathbf{X}$, and a satisfies

$$a \in C(0,\infty) \cap L^1(0,1) \text{ is positive, nonincreasing and log-convex on } (0,\infty) \text{ with } 0 = a(\infty) < a(0^+) \le \infty. \tag{1.2}$$

The resolvents in question are defined formally by

$$\mathbf{u}(t) = \mathbf{U}(t)\mathbf{u}_0 + \mathbf{W}(t)\mathbf{u}_1, \quad \mathbf{V} = \dot{\mathbf{U}}. \tag{1.3}$$

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