JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 2, Number 3, Summer 1990

GLOBAL EXISTENCE IN ONE-DIMENSIONAL NONLINEAR VISCOELASTICITY WITH HEAT CONDUCTION

MORTIN E. GURTIN AND WILLIAM J. HRUSA

Dedicated to our good friend John Nohel on his 65th birthday

1. Introduction. In motions of nonlinear viscoelastic materials the dissipative effects due to memory restrain disturbances of small amplitude, but the destabilizing effects of nonlinearity in the instantaneous response are dominant for disturbances of large amplitude. The difficulties inherent in the study of *general* materials with memory have led to an interest in models that capture the interaction between dissipation and nonlinearity, but are sufficiently simple to allow analysis of corresponding history-value problems. In one space dimension a class of *mechanical* models with this property is based on *single-integral laws*,

$$\sigma(t) = \hat{\sigma}(\varepsilon(t)) + \int_0^\infty \tilde{\sigma}(\varepsilon(t), \varepsilon(t-s), s)\, ds, \tag{1.1}$$

giving the stress $\sigma(t)$ when the strain $\varepsilon(\tau)$ is known at all past times $\tau \le t$.

There is a large literature (cf., e.g., [**13**] and the references cited therein) concerning history-value problems associated with (1.1); in particular, under physically natural assumptions on $\hat{\sigma}$ and $\tilde{\sigma}$, a globally defined smooth solution exists provided the data are smooth and sufficiently close to equilibrium. Similar results have been obtained for rigid heat conductors of single-integral type (cf., e.g., [**1**]) and for thermoelastic heat conductors (cf., e.g., [**8, 14, 16**]). However, to our knowledge there are no general results applicable to nonlinear materials of single-integral type for *deforming bodies under varying temperature*. Our objective here is to study such problems using, as a basis, a thermodynamic theory of viscoelastic materials of single-integral type developed in [**6**]. (A general theory of thermodynamics of materials with memory was developed earlier by Coleman [**2**]. Much of Coleman's theory applies to single-integral laws; however, because single-integral laws have such restricted form, the corresponding set of thermodynamical restrictions is richer than the set originally derived by