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ON THE UNIQUENESS OF SOLUTIONS OF VOLTERRA EQUATIONS

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Dedicated to John Nohel on his 65th birthday

ABSTRACT. It is shown that, except for the obvious case of a Lipschitz continuous nonlinearity and a kernel identically zero close to the origin, the uniqueness of the trivial solution $x(t) \equiv 0$ of the equation $x(t) = \int_0^t k(t-s)g(x(s))\,ds$ depends on both a and g.

1. Introduction and statement of results. Consider the equation

$$x(t) = \int_0^t k(t-s)g(x(s))\,ds, \quad t \geq 0, \tag{1}$$

where k is locally integrable and g is continuous with $g(0) = 0$. It is clear that $x(t) \equiv 0$ is a solution of (1), so the question to be answered is whether there are any other, nontrivial, solutions.

This problem is a special case of the problem of uniqueness of the trivial solution of the equation

$$x(t) = \int_0^t k(t, s, x(s))\,ds, \quad t \geq 0.$$

If the trivial solution is unique one says that k is a Kamke function, and this question is relevant in many problems not directly connected with the uniqueness of solutions of Volterra equations. Although there are definite advantages in treating the more general equation, we will here consider onlyconvolution equations of the form (1).

One of the main tools of the analysis is a comparison principle. Although this result can be found in almost every book on Volterra equations, we state it here in the form that we will need.