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REGULARITY PROPERTIES OF SOLUTIONS OF LINEAR INTEGRODIFFERENTIAL EQUATIONS WITH SINGULAR KERNELS

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Dedicated to John A. Nohel for his sixty-fifth birthday

1. Introduction. Let $\mathbf{H}$ be a Hilbert space with norm $|\cdot|$ and inner product $\langle\cdot,\cdot\rangle$. Let A be a possibly unbounded linear operator in $\mathbf{H}$, and let $a : [0,\infty) \to \mathbf{R}$ be a given locally integrable kernel function. In this note we study properties of solutions of the abstract linear integrodifferential equation

$$u'(t) + a * Au(t) = 0, \quad 0 < t < T, \tag{1}$$

in $\mathbf{H}$. Here the usual convolution notation is employed: $f * g(t) = \int_0^t f(t-s)g(s)\,ds$, if one of the two functions is scalar-valued and the other one is vector-valued.

We want to consider *mild solutions* of (1), i.e., continuous functions $u(\cdot)$ for which $1 * a * u(t) \in D(A)$ for all $0 \le t \le T$ and for which the integrated version

$$u(t) + A(1 * a * u(t)) = u(0) \tag{2}$$

of (1) holds for all $t \in [0,T]$. The goal of this note is to give conditions on the kernel function a under which such mild solutions satisfy an estimate of the form

$$|Au(t)| \le \frac{C}{t^M}|u(0)|$$

for a suitable power M. It will be proved that such an estimate is always true if the derivative a' is integrable and behaves like $-t^{-2\alpha}$ near zero, and that in this case $M = 1/\alpha$ is a suitable exponent.

The following assumptions will be used. The assumptions for the kernel function $a(\cdot)$ are to hold on any finite interval $[0,T]$. A subscript T denotes that the corresponding quantity depends on T.

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