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SOME RESULTS ON NONLINEAR HEAT EQUATIONS FOR MATERIALS OF FADING MEMORY TYPE

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1. Introduction. In this paper we consider a model for the heat conduction for a material covering an n-dimensional bounded set Ω with boundary $\partial\Omega, n = 1, 2, 3$.

(1.1)
$$\begin{cases} \frac{d}{dt}\Big(b_0 u(t,x) + \int_0^t \beta(t-s)u(s,x)\,ds\Big) = c_0\Delta u(t,x), & t > 0, x \in \Omega, \\ u(0,x) = x, \quad x \in \Omega, \end{cases}$$

where $u(t,x)$ is the temperature of the point x at time t (we assume that the temperature is 0 for $x \in \partial\Omega$), b_0 is the *specific heat* and c_0 the *thermal conductivity*. We assume that the specific heat has a term of fading memory type $\int_0^t \beta(t-s)u(s,x)\,ds$, whereas the thermal conductivity is constant. Concerning the kernel β we assume only that it is locally integrable in $[0,\infty[$; this will allow us to consider kernels as $\beta(t) = e^{-\omega t}t^{\alpha-1}, \omega \geq 0, \alpha \in\,]0,1[$.

Model (1.1) (including also a memory term for the thermal conductivity) has been introduced in [**7**] and studied in [**1**] and [**5**].

We write problem (1.1) in abstract form in the Banach space $X = C(\overline{\Omega})$,

$$\begin{cases} \frac{d}{dt}(u(t) + (\beta * u)(t)) = Au(t), & t > 0, \\ u(0) = x, \end{cases} \tag{1.2}$$

where $u(t) = u(t,\cdot)$ and A is the realization in $C(\overline{\Omega})$ of the Laplace operator Δ with Dirichlet boundary conditions.

In order to study (1.2), we assume that A generates an analytic semigroup and that β is Laplace transformable with Laplace transform $\hat{\beta}(\lambda)$ analytic in a sector $S_{\omega,\theta} = \{\lambda \in \mathbf{C} \setminus \{0\} : |\arg(\lambda - \omega)| < \theta\}$ with $\omega \in \mathbf{R}$ and $\theta \in\,]\pi/2, \pi[$. Then the Laplace transform $\hat{u}(\lambda)$ of u is given formally by

$$\hat{u}(\lambda) := F(\lambda)x = R(\lambda + \lambda\hat{\beta}(\lambda), A)x. \tag{1.3}$$

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