JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 2, Number 3, Summer 1990

PARAMETER IDENTIFICATION IN A VOLTERRA EQUATION WITH WEAKLY SINGULAR KERNEL

DENNIS W. BREWER AND ROBERT K. POWERS

This paper is dedicated with gratitude to my thesis advisor, John A. Nohel, on the occasion of his sixty-fifth birthday.

ABSTRACT. We consider identification of parameters in a Volterra integrodifferential system with a weakly singular kernel. Such kernels arise in fractional derivative damping models of viscoelastic materials. The Volterra equation is cast in a semigroup setting to establish results on the differentiability of the solution with respect to a parameter. These results are needed for convergence of the identification algorithm. Numerical results are presented.

1. Introduction. In this paper we consider the identification of parameters in a Volterra integrodifferential equation with a singular kernel. The equation of interest has the form

$$(1.1)\quad \begin{cases} \dot{w}(t) = Mw(T) + \int_{-\infty}^{t} K(t-s,p)w(s)\,ds + F(t), & t \geq 0, \\ w(0) = \eta,\ w(s) = \phi(s), & s < 0, \end{cases}$$

where M is an $n \times n$ constant matrix, $\eta \in \mathbf{R}^n$, $\phi \in L^1(-\infty, 0; \mathbf{R}^n)$ and $K(\cdot, p)$ is an $n \times n$ singular kernel depending on a parameter p contained in an admissible parameter set. We are particularly interested in a kernel function of the form

$$g(s,p) = \frac{\gamma e^{-\beta s}}{\Gamma(1-\alpha)s^{\alpha}}, \quad s > 0,$$

where $\Gamma(\cdot)$ denotes the gamma function, $p = (\alpha, \beta, \gamma) \in \mathbf{R}^3$ with $0 \leq \alpha < 1$ and $\beta, \gamma > 0$. Such kernels arise in the study of fractional derivative models of viscoelastic structures. For a more complete discussion of the origins of this kernel and the viscoelastic models, we

This research was supported by the Air Force Office of Scientific Research under Grant AFOSR-89-0472.