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GLOBAL EXISTENCE OF SMOOTH SHEARING MOTIONS OF A NONLINEAR VISCOELASTIC FLUID

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Dedicated to John Nohel on the occasion of his sixty-fifth birthday.

1. Introduction. The aim of this note is to establish global existence of smooth rectilinear shearing motions of incompressible nonlinear viscoelastic fluids of $\mathrm{K}-\mathrm{BKZ}$ type. These fluids, which are described by constitutive relations of integral type, were introduced independently by Kaye [**7**] and Bernstein, Kearsley and Zapas [**1**]. Global existence theorems have been obtained previously by Kim [**8**] and by Renardy, Hrusa and Nohel [**10**, Section IV.5]. Kim discusses a situation in which the fluid occupies all of $\mathbf{R}^3$ and the nonlinearity in the constitutive equation has a special form. Renardy, Hrusa and Nohel study spatially periodic three-dimensional motions with a general nonlinearity in the constitutive equation. In [**8**] and [**10**] the initial data are assumed to be smooth and small and the kernel of the constitutive relation is assumed to be smooth on $[0,\infty)$.

The equation of motion that we shall consider is

(1.1)
$$u_{tt}(x,t) = \int_0^\infty a'(s)g(u_x(x,t)-u_x(x,t-s))_x\,ds+f(x,t), \quad x\in B,\ t\geq 0,$$

where subscripts x and t indicate partial derivatives and a' denotes the derivative of a. Here the unknown u is a component of the displacement, f is a forcing function, $a:[0,\infty)\to\mathbf{R}$ and $g:\mathbf{R}\to\mathbf{R}$ are smooth constitutive functions, and $B\subset\mathbf{R}$ is an interval. It follows from symmetry considerations that g is an odd function, i.e., $g(-\xi)=-g(\xi)$ for all $\xi\in\mathbf{R}$. We refer to Coleman and Noll [**4**] and to Sections 2 and 3 of Coleman and Gurtin [**3**] for a general discussion of shearing motions of incompressible viscoelastic fluids. The monograph [**10**] contains relevant information on $\mathrm{K}-\mathrm{BKZ}$ fluids as well as derivation of (1.1) for rectilinear shearing motions.