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DISSIPATIVE BOUNDARY CONDITIONS FOR ONE-DIMENSIONAL WAVE PROPAGATION

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To John Nohel for his sixty-fifth birthday

1. Introduction. There has been a great deal of work in recent years on evolution equations which contain memory terms. The most interesting situation occurs for wave propagation in elastic materials. One starts with a model which conserves energy and then modifies it by including a memory term which produces damping (*viscoelasticity*). John Nohel has been a major figure in these studies and the results are summarized in his book with Hrusa and Renardy [**7**].

The present paper is concerned with a closely related but slightly different idea. Here we maintain an energy conserving equation but produce damping through boundary conditions. Let us describe the problem and then we will indicate why it is of interest.

We deal with one-dimensional longitudinal motions of a bar which has uniform cross section but may be inhomogeneous. The basic balance law, in the absence of body forces, is

$$\rho u_{tt} = \sigma_x, \tag{1.1}$$

where ρ is density, u displacement and σ stress. The specific problem we consider is this:

$$(P(\varphi,\psi)) \qquad \begin{gathered} \rho(x)u_{tt}(x,t) = (\mu(x)u_x(x,t))_x, \quad 0 < x < L, \\ u(x,0) \equiv u_t(x,0) \equiv 0 \\ u(0,t) = \varphi(t), \quad \mu(L)u_x(L,t) = \mathcal{F}\,[u^t(L,\cdot)] + \psi(t). \end{gathered}$$

Here, φ and ψ are given and $\mathcal{F}$ denotes a functional of the history

$$u^t(L,\tau) = u(L,t-\tau).$$

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