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MAXIMAL REGULARITY AND GLOBAL WELL-POSEDNESS FOR A PHASE FIELD SYSTEM WITH MEMORY

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ABSTRACT. In this paper we obtain global strong well-posedness for a phase field system with memory and relaxing chemical potential in an L_p-setting, employing maximal regularity tools. The global well-posedness result is obtained by an energy estimate, provided that the space dimension n is less than 3.

1. Introduction. Let $\Omega \subset \mathbf{R}^n$ be a bounded domain with smooth boundary $\partial\Omega$ and let $J = [0,T]$, $T > 0$, be an interval. We consider the following system

$$
(PFM)\begin{cases}
u_t + \dfrac{l}{2}\,\phi_t = \displaystyle\int_{-\infty}^{t} a_1(t-s)\Delta u(s)\,ds & \text{in } J\times\Omega;\\
\tau\phi_t = \displaystyle\int_{-\infty}^{t} a_2(t-s)\Big[\xi^2\Delta\phi + \frac{\phi-\phi^3}{\eta} + u\Big](s)\,ds & \text{in } J\times\Omega;\\
\mathbf{n}\cdot\nabla u = \mathbf{n}\cdot\nabla\phi = 0 & \text{on } J\times\partial\Omega;\\
u(0,x) = u_0(x),\ \ \phi(0,x) = \phi_0(x) & \text{in } \Omega.
\end{cases}
$$

The phase field system with memory (PFM) was first proposed in [**9**] as a phenomenological model to describe phase transitions in the presence of a slowly relaxing internal variable. Later Novick-Cohen [**6**] obtained a global weak solution of (PFM), by means of the Galerkin method and energy estimates.

Our goal here is to obtain global well-posedness of (PFM) in the strong sense in an L_p-setting. Assuming enough regularity of the kernels, we may apply a recent result in the theory of Volterra equations,

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