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# WEAK SOLUTIONS OF THE EXTERIOR BOUNDARY VALUE PROBLEMS OF PLANE COSSERAT ELASTICITY

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ABSTRACT. In this paper we formulate exterior Dirichlet and Neumann boundary value problems of plane Cosserat elasticity in Sobolev spaces, show that these problems are well-posed and find the corresponding weak solutions in terms of integral potentials.

**1. Introduction.** The theory of micropolar (Cosserat) elasticity [**4**] has been developed by Eringen to account for discrepancies between the classical theory and experiments when the effects of material microstructure were known to significantly affect the body's overall deformation, for example, in the case of granular bodies with large molecules (e.g. polymers) or human bones, see [**7**–**10**]. Significant progress has been achieved in this direction for the last 30 years (see [**12**] for a review of works in this area and an extensive bibliography), but investigations mainly have been confined to the case of boundary value problems for domains bounded by sufficiently smooth curves. For example, three-dimensional problems of Cosserat elasticity have been formulated in a rigorous setting and solved by means of methods of the potential theory by Kupradze in [**6**].

In [**5**, **13**–**15**], the corresponding boundary value problems for plane deformations of a micropolar homogeneous, linearly elastic solid were shown to be well posed and subsequently solved in a rigorous setting using the boundary integral equation method. Unfortunately, consideration of these problems in the space $L^2$ setting requires that we impose strict conditions on the curve which represents the boundary of the domain. To be precise, this curve must be expressed in terms of a twice differentiable function. If this condition is not satisfied, i.e., the boundary is not smooth enough or the domain contains cracks, the method presented in [**5**, **13**–**15**] fails to produce acceptable results. To overcome this difficulty it seems reasonable to formulate the corresponding

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