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ON A BOUNDARY VALUE PROBLEM IN SUBSONIC AEROELASTICITY AND THE COFINITE HILBERT TRANSFORM

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ABSTRACT. We study a boundary value problem in subsonic aeroelasticity and introduce the *cofinite Hilbert transform* as a tool for solving an auxiliary linear integral equation on the complement of a finite interval on the real line $\mathbf{R}$.

1. Introduction. We consider the linearized subsonic inviscid compressible flow equation in two dimensions [**2, 3**]

$$a^2\left(1-M^2\right)\frac{\partial^2\phi}{\partial x^2}+a^2\frac{\partial^2\phi}{\partial z^2}=\frac{\partial^2\phi}{\partial t^2}+2Ma\,\frac{\partial^2\phi}{\partial t\partial x}, \tag{1}$$

where a is the speed of sound, M is the Mach number ($M=U/a<1$, where U is the free stream velocity), and $\phi(x,z,t)$ is a small disturbance velocity potential, considered on

$$\mathbf{R}^2_+\times\overline{\mathbf{R}_+}=\{(x,z,t):-\infty<x<\infty,\ 0<z<\infty,\ 0\le t<\infty\}.$$

This velocity potential is assumed to satisfy the boundary conditions:

- flow tangency condition

$$\frac{\partial\phi}{\partial z}(x,0,t)=w(x,t),\quad |x|<b, \tag{2}$$

where b is the "half-chord," and w is the given normal velocity of the wing, without loss of generality we will assume in what follows that $b=1$,

- "*strong Kutta-Joukowski condition*" for the acceleration potential

$$\begin{gathered}\psi(x,z,t):=\frac{\partial\phi}{\partial t}+U\,\frac{\partial\phi}{\partial x},\\ \psi(x,0,t)=0\quad\text{for}\quad 1<|x|<A\quad\text{for some}\quad A>1,\end{gathered} \tag{3}$$

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